Unit 6A → Exponential Functions and Equations

Module 12 - Sequences & Series

Definitions: A sequence is an ordered list of numbers

A series is the sum of a number of terms.

12.1 Notes

1) Consider the sequence 2, 5, 8, ...

Arithmetic sequence - also known as a discrete linear function. Consecutive terms have a common difference, d.

a) Show the common difference is $d = 3$

$d = 5 - 2 = 3$ $\quad d = 8 - 5 = 3$

b) Find the 7th term (let 2 = 1st term)

2, 5, 8, 11, 14, 17, 20

7th term = 20

To define sequences Recursively, identify the starting value and a rule to find the next term.

Starting value $f(1) = a$

1st term ↑ ↖ value of 1st term

our example: 2, 5, 8... $f(1) = 2$

Rule $f(n) = f(n-1) + d, \quad n \geq 2$ ← use this rule from 2nd term on

$f(n)$: next term ↑; $f(n-1)$: previous term ↓; d: common difference ↑

our example: $f(n) = f(n-1) + 3, \quad n \geq 2$

Sometimes the 1st term is considered a starting value and is called the 0-term

$f(0) = a$, $f(n) = f(n-1) + d$, $n \geq 1$

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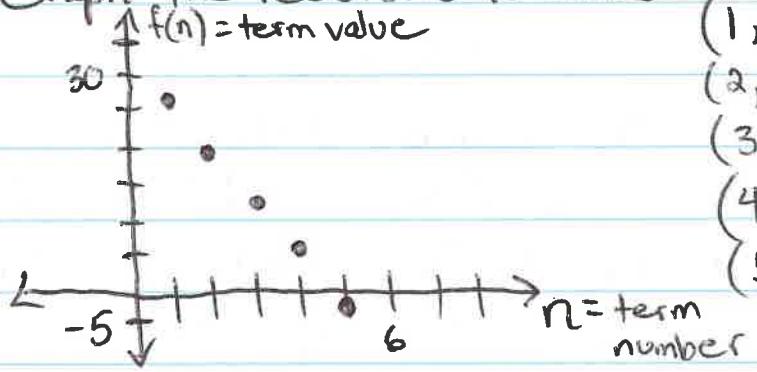
2) Given the recursive formula,

$f(1) = 26$, $f(n) = f(n-1) - 7$, $n \geq 2$

a) Find the first 5 terms

26, 19, 12, 5, -2, ...

b) Graph the recursive formula



(1, 26)
(2, 19)
(3, 12)
(4, 5)
(5, -2)

* Discrete points are not connected, not continuous!

3) Consider $\overset{0}{-3}, \overset{1}{2}, \overset{2}{7}, \overset{3}{12}, \ldots$

a) Define recursively using $f(0) = -3$

$f(n) = f(n-1) + 5$, $n \geq 1$

b) Find the 6th term

$f(6) = f(5) + 5$

$f(5) = f(4) + 5$

$f(4) = f(3) + 5$

$f(4) = 12 + 5 = 17$

$f(5) = 17 + 5 = 22$

$f(6) = 22 + 5 = 27$

Sequences can also be defined Explicitly

Explicit equations find any term without having to know the previous term.

4) Consider 6, 4, 2, 0, ... Let 6 be $f(0)=6$

a) Find the 50^{th} term.

Write an explicit equation

$$f(n) = a + dn$$
$$f(50) = 6 - 2(50)$$
$$= 6 - 100$$
$$= -94$$

b) which term will have a value of -216?

$$-216 = 6 - 2n$$
$$-6 \quad -6$$
$$\frac{-222}{-2} = \frac{-2n}{-2}$$
$$n = 111$$

5) Let $f(1) = 10$, $f(n) = f(n-1) + 6$, $n \geq 2$

a) Find the 60^{th} term by writing an explicit formula.

$$f(n) = a + d(n-1)$$
$$f(60) = 10 + 6(60-1)$$
$$f(60) = 10 + 6(59)$$
$$= 364$$